

KINETICS OF HEATING DISPERSE MATERIALS
DURING DRYING IN A VIBROFLUIDIZED LAYER

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Approximate solutions of the main equation of heat and mass transfer for a periodic and continuous process of drying in a vibrofluidized layer are given, and compared with experimental data; a Reh binder number is used to establish the connection between the temperature of the material and the moisture content.

Drying in a vibrofluidized layer in a number of cases can successfully compete with other modern methods of drying moist disperse materials, especially with drying in a fluidized layer [1, 2]. It is especially useful to dry finely dispersed adhering materials in a vibrofluidized layer, and also materials which cannot be removed from the apparatus.

As is already known [3, 4], in a number of cases it is useful to supply conductive heat to the moist material which is in a vibrofluidized state, especially when direct contact of the material with the heat carrier is unsuitable. Research on the drying process and the kinetics of heating disperse materials in a vibrofluidized layer when heat is supplied by conduction was carried out in [3, 7]. In these works, however, the temperature of the heating surface was constant. In practice it is possible to use different methods of introducing heat to the heating surface of the vibro-dryer: heating the surface with superheated vapor, electric heating, induction heating, hot gases etc. In these cases the temperature of the heating surface can be constant, or variable. Moreover, in the work [3] approximate analytical calculation of the kinetics of heating a moist disperse material was examined only in the case of a constant drying rate.

In connection with this we have carried out research on the kinetics of drying and heating in a vibrofluidized layer in the case of conductive supply of heat for periods of constant and falling drying rate, with a varying, and constant temperature of the heating surface. A finely disperse sodium chloride, whose fractional composition is given in Table 1, was selected as the material to be investigated. The mean diameter of the particles calculated according to the formula $d_{mn} = \sum d_i \Delta g_i / \sum \Delta g_i$, is $d_{mn} = 3.66 \cdot 10^{-4}$ m. Here $d_i = (d_1 + d_2) / 2$, where d_1 and d_2 are the extreme dimensions of the meshes for a given fraction; Δg_i is the quota of a given fraction in weight %. The initial moisture content of the salt in the experiments was $\bar{u}_1 = (4-6) \cdot 10^{-2}$ kg/kg, and the final content was $\bar{u}_2 = (0.05-0.1) \cdot 10^{-2}$ kg/kg.

The experimental research was carried out on an experimental apparatus whose layout is shown in Fig. 1. The moist material is loaded into a chamber 1, which has an electric heater 11 built into the bottom. The supply of the electric heater is obtained from an ac circuit through the laboratory autotransformer 13. The vibrating table 2, on which the chamber with the material is placed is caused to oscillate by a dc electric motor 4. The supply of the electric motor is obtained from the rectified VAZ-50-125. The frequency of the vibrations of the vibrating table in a range of 0-50 Hz is smoothly varied by the speed of rotation of the motor, to which a regulated dc voltage was supplied. The speed of rotation of the motor was

TABLE 1. Screening analysis of sodium chloride

Dimensions of the apertures, mm	0,630	0,400	0,355	0,315	0,250	0,200	0,150	0,100	0,063	0,063
Residue on the screen, %	2,80	30,2	12,7	14,2	20,9	8,10	5,70	4,30	0,80	0,30

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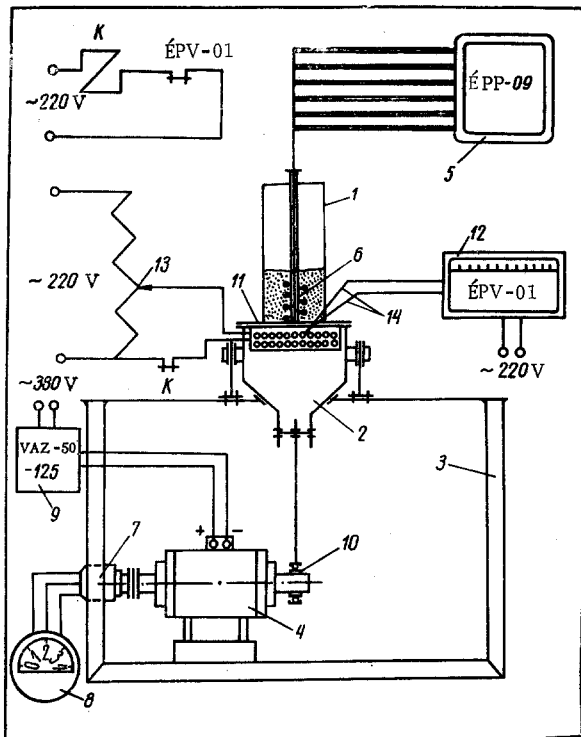


Fig. 1. Main layout of the experimental vibro-dryer with conductive heat supply: 1) chamber for the material being dried; 2) vibrating table; 3) bed plate; 4) dc electric motor; 5) electronic automatic potentiometer ÉPP-0.9-M2; 6) thermal probe; 7) electrotachometer transducer; 8) secondary device of the electrotachometer; 9) rectifier VAZ-50-125; 10) eccentric with variable eccentricity; 11) electric heater; 12) electronic potentiometer ÉPV-0.1; 13) laboratory autotransformer; 14) chromel - copel thermo-couple.

fixed by the electric tachometer 7, 8. The amplitude of the vibrations varied step by step from 0-5 mm by using an eccentric mechanism with varying eccentricity. The experiments were carried out with a heating surface temperature which was constant and varied with time. In order to maintain the determined constant temperature of the surface, automatic regulation was provided using mercury changeover switches built into the electronic potentiometer 12 of the type ÉPV-0.1. The variable temperature of the surface was established according to a given previously established law by using a controlling measuring apparatus. The temperature of the heating surface was recorded by the potentiometers 12 and 5, by using chromel - copel thermocouples which had been connected to them and peened into the surface. The temperature of the salt over the height of the layer and during the drying process was measured by a thermal probe which consisted of 10 chromel - copel thermocouples with a junction diameter of 0.3 mm, connected to the potentiometer 5. The thermal probe was located at the center of the working chamber and rigidly attached to the latter. The parameters of the vibration varied in the following manner: the amplitude was within the limits $A^* = 1-5$ mm, and the frequency was $f = 10-50$ Hz. The temperature of the heating surface was 150-300°C, and the height of the dense layer was $h_{de} = 20-90$ mm. The working chamber had a rectangular cross section of 55 × 240 mm, and its height was 350 mm.

The previously moistened and weighed salt was loaded into the working chamber which vibrated at a given amplitude and frequency, and it was dried to a constant weight. During the course of the experiment the temperature of the heating surface of the material and the frequency of vibration were continuously adjusted.

The experimental data according to the kinetics of drying were treated by a method proposed by Lykov [9]:

$$-\frac{d\bar{u}}{d\tau} = \kappa N (\bar{u} - u_p). \quad (1)$$

The magnitude N in expression (1) depends on the drying condition (t^* , φ , A^* , f),[†] and the relative drying factor κ depends on the properties of the material which is being dried and on its initial moisture.

Some experimental drying curves and the variation of the temperature of the material with respect to time are represented by points shown in Fig. 2. The significant part of the process of drying the small crystal sodium chloride in a vibrofluidized layer takes place at a constant speed. During this period the main quantity of moisture (approximately from $5.5 \cdot 10^{-2}$ to $0.7 \cdot 10^{-2}$ kg/kg) is removed in all drying conditions. This demonstrates the intensity with which the drying speed decreases, the remaining quantity of

[†] For $t = \text{const}$, $t^* = t$, for $t = f(\tau)$, $t^* = t_{\text{max}}$.

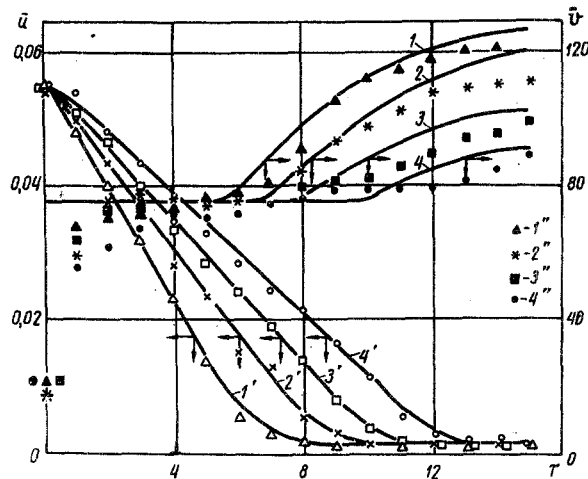


Fig. 2. Drying curves and temperature curves ($t = 165^{\circ}\text{C}$, $h_{de} = 44$ mm, $f = 28.3$ Hz): 1, 1', 1'' $A^* = 4.5$ mm; 2, 2', 2'' 3.5 mm; 3, 3', 3'' 2.5 mm; 4, 4', 4'' 1.5 mm.

moisture is removed (up to $\sim(0.1-0.05) \cdot 10^{-2}$ kg/kg), but both periods are approximately the same with respect to time. It has also been established experimentally that the process of drying intensifies with increase in the amplitude of vibration. Another parameter of vibration, frequency, in the investigated range influences the drying rate to a lesser extent; hence, for example, when $f = 21.4$ Hz, $A^* = 4.5$ mm $\frac{d\bar{u}}{d\tau} = -N = 7.4 \cdot 10^{-3}$ kg/kg/min, and where $A^* = 1.5$ mm and with the same frequency $N = 5.1 \cdot 10^{-3}$ kg/kg/min, i.e., the drying rate in the first period increases hence approximately 1.5 times. When $A^* = 2.5$ mm and the variations of the frequency from $f_1 = 40$ Hz to $f_2 = 14.7$ Hz the drying rate varies from $7.6 \cdot 10^{-3}$ to $6.7 \cdot 10^{-3}$ kg/kg/min, that is, 1.1 times. These experiments were carried out at a constant temperature of the heating surface $t = 165^{\circ}\text{C}$, with a height of the dense layer $h_{de} = 44$ mm and $u_1 = 5.5 \cdot 10^{-2}$ kg/kg.

This influence of the amplitude and frequency of the vibrations corresponds with the data which are known from the literature [3, 7]. It is established by research on the behavior of the material in a vertically vibrating vessel [7] that when the amplitude of the vibration increases in a certain range, in the case of constant frequency, the mixing of the particles in the layer improves. The influence of the variation in the frequency of the vibrations in the case of invariable amplitude has less effect on the intensity of mixing. The influence of the amplitude and frequency of the vibrations on the drying rate in the first period can evidently be explained by this. It is natural that in order to maintain a constant temperature of the heating surface the quantity of heat supplied in a unit of time must be different.

We will examine the kinetics of heating the moist material during drying in a vibrofluidized layer in greater detail. We will adopt the following known assumptions [3, 5, 6]: constancy of the temperature of the material which is being dried over the height of the vibrofluidized layer, the gradientless heating of the particles of material, the invariability of the effective coefficient of heat exchange from the heating surface to the material which is being dried in which this coefficient also takes into account the transfer of heat by convection and by radiation, and also heat exchange between the particles in the vibro-liquid layer, and the neglecting of heat losses into the surrounding medium.

The equation of the thermal balance when applied to the periodic process of drying (Fig. 3a) in a vibrofluidized layer similar to [8] is written in the form

$$\alpha_{ef} F (t - \bar{\theta}) d\tau = Gc_m \left(1 + \frac{c_l}{c_m} \bar{u} \right) d\bar{\theta} - rGd\bar{u}. \quad (2)$$

Hence it must be noted that a similar balancing equation

$$\alpha_{ef} F_{sp} (t - \bar{\theta}) dx = G^*c_m \left(1 + \frac{c_l}{c_m} \bar{u} \right) d\bar{\theta} - rG^*d\bar{u} \quad (3)$$

will also describe the process of drying a moist disperse material in a continuous action vibro-dryer (Fig. 3b) with the difference that instead of the time differential $d\tau$ a path differential must be substituted into the

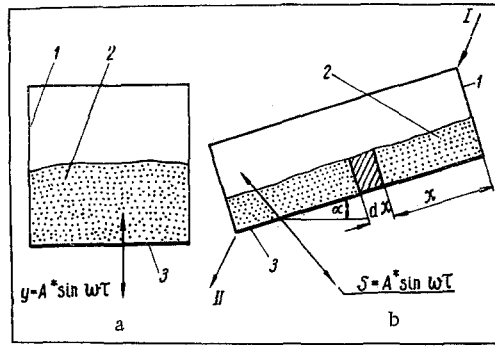


Fig. 3. Calculated diagrams of a) periodic-action vibro-dryers, and b) continuous vibro-dryers, with conductive heat supply: 1) vibrating vessel (tray); 2) material which is being dried; 3) heating surface; I) moist material; II) dried material.

left-hand part of the equation, i.e., to convert the mean speed of movement of the material into a time differential $dx = \bar{v}d\tau$, instead of the surface heating F , a specific surface F_{sp} must be substituted which comes into the unit path of displacement of the treated material, and instead of the mass of the charge, there should be a mass output of the dryer, in terms of dry material.

With constant temperature of the heating surface

$$t = \text{const.} \quad (4)$$

In the case of heating of the surface with gases the variation in the temperature of the heating surface with respect to time is expressed with sufficient accuracy satisfactorily by the expression

$$t = t_k + (t_{in} - t_k) \exp(-\rho\tau). \quad (5)$$

We will examine the solution of the equation (2) in the case of the condition (4) for the period of decreasing drying speed. It is known [9] that for this drying period the expression (1) is applicable. By integrating (1) from 0 to τ and from u_{cr1} to u we have

$$\bar{u} = u_p + (\bar{u}_{cr1} - u_p) \exp(-\kappa N\tau). \quad (6)$$

By substituting (4), and (6) into (2), we will obtain

$$\alpha_{ef} F(t - \bar{\theta}) d\tau = Gc_m \left\{ 1 + \frac{c_l}{c_m} [u_p + (\bar{u}_{cr1} - u_p) \exp(-\kappa N\tau)] \right\} d\bar{\theta} + rG\kappa N (\bar{u}_{cr1} - u_p) \exp(-\kappa N\tau) d\tau. \quad (7)$$

We will define:

$$\frac{\alpha_{ef} F}{Gc_m} = A; \quad (8)$$

$$\frac{r\kappa N (\bar{u}_{cr1} - u_p)}{c_m} = B; \quad (9)$$

$$1 + \frac{c_l}{c_m} u_p = E; \quad (10)$$

$$\frac{c_l}{c_m} (\bar{u}_{cr1} - u_p) = D. \quad (11)$$

Taking into account (8)-(11), equation (7) is converted to the form

$$\frac{d\bar{\theta}}{d\tau} + \frac{A}{E + D \exp(-\kappa N\tau)} \bar{\theta} - \frac{At - B \exp(-\kappa N\tau)}{E + D \exp(-\kappa N\tau)} = 0. \quad (12)$$

The solution of equation (12) is

$$\bar{\theta} = \frac{y^a}{(E + Dy)^a} \left[C - \frac{At}{\kappa N} \int \frac{(E + Dy)^{a-1}}{y^{a+1}} dy + \frac{B}{\kappa N} \int \frac{(E + Dy)^{a-1}}{y^a} dy \right], \quad (13)$$

where

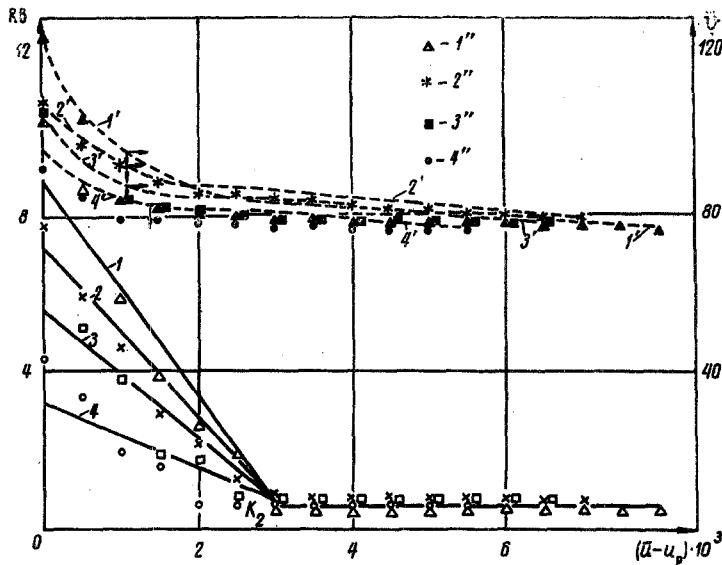


Fig. 4. Relationship between the Reh binder number Rb and the temperature of the material, and the moisture content ($t = 165^\circ\text{C}$, $h_{de} = 44$ mm). Parameters of vibration see Fig. 2.

$$y = \exp(-\kappa N \tau); \quad (14)$$

$$a = \frac{A}{E\kappa N}. \quad (15)$$

As a rule, for many materials, especially for crystalline disperse materials the equilibrium moisture is close to zero. Therefore $E = 1$ can be adopted without significant error. We will also examine the product Dy : the magnitude D , in accordance with (11), is less than unity and will fluctuate from hundredths to thousandths part of unity; the magnitude y in the period of falling speed decreases from several tenths to ten thousandths part of unity. As a result it is found that the product $Dy \ll 1$, and therefore it is possible to take $E + Dy = \sim 1$. Hence instead of (13) we will have

$$\bar{\vartheta} = y^a \left[C - \frac{At}{\kappa N} \int \frac{dy}{y^{a+1}} + \frac{B}{\kappa N} \int \frac{dy}{y^a} \right] = Cy^a + \frac{At}{\kappa Na} + \frac{By}{\kappa N(1-a)} \quad (16)$$

From the initial conditions $\bar{\vartheta}|_{y=y_1} = \bar{\vartheta}_1$ we will determine the arbitrary integration constant C :

$$C = \frac{\bar{\vartheta}_1 - \frac{At}{\kappa Na} - \frac{B}{\kappa N(1-a)} y_1}{y_1^a}, \quad (17)$$

where $y_1 = \exp(-\kappa N \tau_1)$, and τ_1 is the drying time in the period of constant speed up to the critical moisture content.

Taking (17) into account, the expression (16) is converted to the form

$$\bar{\vartheta} = \bar{\vartheta}_1 \left(\frac{y}{y_1} \right)^a - \frac{At}{\kappa Na} \left[\left(\frac{y}{y_1} \right)^a - 1 \right] - \frac{B}{\kappa N(1-a)} \left(\frac{y^a}{y_1^{a-1}} - y \right). \quad (18)$$

Hence the final form of the solution for former initial conditions for the period of decreasing speed has the form (18).

From analysis of equation (2) by condition (4) for a period of constant drying speed where $\bar{\vartheta} = \text{const}$, i.e., $d\bar{\vartheta}/d\tau = 0$, it is possible to obtain a relationship for calculating the mean temperature of the material for this period with respect to the height of the layer

$$\bar{\vartheta} = t - \frac{B_1}{A}, \quad (19)$$

where

$$B_1 = \frac{rN}{c_M} \quad (20)$$

The solution of equation (2) in the case of variable temperature of the heating surface, i.e., for condition (5), for the period of decreasing drying speed taking into account the previous designations is

$$\bar{\vartheta} = \left(\frac{y}{E + Dy} \right)^a \left[C - \frac{At_k}{\kappa N} \int \frac{(E + Dy)^{a-1}}{y^{a+1}} dy - \frac{A(t_{in} - t_k)}{\kappa N} \int \frac{(E + Dy)^{a-1}}{y^{a - \frac{p}{\kappa N} + 1}} dy + \frac{B}{\kappa N} \int \frac{(E + Dy)^{a-1}}{y^a} dy \right] \quad (21)$$

The expression (21) taking into account the previous premises can be written in the following form:

$$\bar{\vartheta} = y^a \left[C - \frac{At_k}{\kappa N} \int \frac{dy}{y^{a+1}} - \frac{A(t_{in} - t_k)}{\kappa N} \int \frac{dy}{y^{a - \frac{p}{\kappa N} + 1}} + \frac{B}{\kappa N} \int \frac{dy}{y^a} \right] \quad (22)$$

In accordance with experimental data, it is possible to assume that $a \approx p/\kappa N$, and then

$$\bar{\vartheta} = y^a \left[C - \frac{At_k}{\kappa N} \int \frac{dy}{y^{a+1}} - \frac{A(t_{in} - t_k) - B}{\kappa N} \int \frac{dy}{y^a} \right] = Cy^a + \frac{At_k}{\kappa Na} + \frac{A(t_{in} - t_k) - B}{\kappa N(a-1)} y \quad (23)$$

From the initial conditions $\bar{\vartheta}|_{y=y_1} = \bar{\vartheta}_1$ we will determine the integration constant C:

$$C = \frac{\bar{\vartheta}_1}{y_1^a} - \frac{At_k}{\kappa Nay_1^a} - \frac{A(t_{in} - t_k) - B}{\kappa N(a-1)y_1^{a-1}} \quad (24)$$

The expression (23) for determining the temperature of the material taking (24) into account assumes the form

$$\bar{\vartheta} = \bar{\vartheta}_1 \left(\frac{y}{y_1} \right)^a - \frac{At_k}{\kappa Na} \left[\left(\frac{y}{y_1} \right)^a - 1 \right] - \frac{A(t_{in} - t_k) - B}{\kappa N(a-1)} \left(\frac{y^a}{y_1^{a-1}} - y \right) \quad (25)$$

Figure 2 gives the temperature curves (continuous lines), calculated according to the formulae (18)-(19). The experimental data (points) are also plotted. In all cases the calculated curves are somewhat higher than the experimental curves, in which this deviation does not exceed 10%.

Hence, the ratios (18), (19), and (25) establish the connection between the temperature of the material, and the time in the drying process. However, in order to calculate the drying process the relationship between the temperature and the moisture content of the material is quite important [10]. Such a connection is established by using the Reh binder number for approximate calculations in the period of the decreasing drying speed

$$Rb = \frac{c}{r} \left(\frac{d\bar{\vartheta}}{du} \right) \quad (26)$$

A well-defined relationship between the temperature of the material and its moisture content can be obtained directly from expressions (28), and (25), substituting τ into these (into magnitude y) from equation (1) through \bar{u} . However, as can easily be seen, expressions which are cumbersome and unsuitable for calculation would have been obtained. Therefore in order to establish a connection between the temperature and the moisture content of the material we used a different approach using the Rb.

The experiments which we carried out on drying fine crystal sodium chloride in a vibrofluidized layer and their subsequent treatment according to (26) give the following relationships, set out in Fig. 4 for a given case:

$$\begin{aligned} Rb &= -0.6 \quad \text{for} \quad \bar{u}_{kr_2} \leq \bar{u} \leq \bar{u}_{kr_1}, \\ Rb &= kT [0.8(\bar{u} - u_p) \cdot 10^3 - 2.0] \quad \text{for} \quad u_p \leq \bar{u} \leq \bar{u}_{kr_2}. \end{aligned} \quad (27)$$

As numerous experimental investigations have shown [11], in a general case

$$Rb = \frac{d\bar{\vartheta}}{d\bar{u}} \frac{c_0 + c_L \bar{u}}{r} = Lf(\bar{u}),$$

where the experimental factor L does not depend on the moisture content. Hence we have

$$\int_{\bar{\vartheta}_{ef}}^{\bar{\vartheta}} dz = Lr \int_{\bar{u}_{ef}}^{\bar{u}} \frac{f(\xi)}{c_0 + c_L \xi} d\xi \quad (28)$$

However, in a number of cases instead of the local value $c = c_0 + c_1 \bar{u}$ an averaged value of volume and the time of the process (or part of the process) is taken, with specific heat \bar{c} . We will then obtain instead of (28), as in (11)

$$\bar{\theta} = \bar{\theta}_{ef} + L \frac{r}{c} \int_{\bar{u}_{ef}}^{\bar{u}} f(\zeta) d\zeta. \quad (29)$$

Substituting (27) into (29) we obtain correspondingly:

$$\bar{\theta} = \bar{\theta}_1 + \frac{0.6r}{c} (\bar{u}_{kr1} - \bar{u}) \quad \text{for } \bar{u}_{kr2} \leq \bar{u} \leq \bar{u}_{kr1}, \quad (30)$$

$$\bar{\theta} = \bar{\theta}_1 + \frac{0.4 \cdot 10^3 r k T}{c} [(\bar{u} - u_p)^2 - (\bar{u}_{kr2} - u_p)^2] + \frac{2rkT}{c} (\bar{u}_{kr2} - \bar{u}) + \frac{0.6r}{c} (\bar{u}_{kr1} - \bar{u}_{kr2}) \quad (31)$$

where

$$u_p \leq \bar{u} \leq \bar{u}_{kr2}.$$

In (31) the relationship (30) is taken into account.

The curves (hatched), calculated according to (30) and (31), are given in Fig. 4. Experimental points are also plotted. The discrepancy hence is not more than 8%.

In conclusion, we note that it is possible to obtain a ratio for determining the heating surface (or the magnitude α) from the expression (20), and hence the remaining parameters of the process must be known.

For calculations according to the formulae given in the work it is necessary to know the relationship for determining α_{ef} and N . The authors propose to publish the results of research for determination of these magnitudes in a separate report.

NOTATION

A^*	is the amplitude of vibration, mm;
c	is the specific heat J/kg · deg;
f	is the frequency of vibration, Hz;
g	is the gravitational acceleration, m/sec ² ;
G	is the mass of charge, kg;
h_{de}	is the height of dense layer of charge, mm;
r	is the latent heat of evaporation, J/kg;
t	is the temperature of the heating surface, °C;
\bar{u}	is the mean moisture content of the material with respect to the volume, kg · moist/kg dry material;
u_1	is the initial moisture content, kg · moist/kg dry material;
α_{ef}	is the effective coefficient of heat exchange, W/m ² · deg;
$\bar{\theta}$	is the mean temperature of the material with respect to the volume;
κ	is the relative drying factor;
τ	is the time, min;
ω	is the circular frequency, sec ⁻¹ ;
Rb	is the Reh binder number;
K	is the factor of the working condition ($K = A^* \omega^2 / g$);
T	is the dimensionless temperature [$T = (t - \vartheta_0) / (\vartheta_0 + 273)$].

Subscripts

l	denotes the liquid;
cr_1	denotes the first critical state;
cr_2	denotes the second critical state;
k	denotes the final value;
M	denotes the material;
in	denotes the initial value;
o	denotes the condition of absolutely dry body;
p	denotes the equilibrium state;
Φ	fixed value.

LITERATURE CITED

1. S. S. Zabrodskii, Hydrodynamics and Heat Exchange in a Fluidized Bubbling Layer [in Russian], Gosenergoizdat, Moscow - Leningrad (1963).
2. Z. A. Kats and A. P. Rysin, *Konservnaya i Ovoshchesushil'naya Promyshlennost'*, No. 1 (1965).
3. V. I. Syroedov, Graduate Thesis, Moscow Technological Institute of the Food Industry (1967).
4. V. A. Sheiman, A. S. Zelepuga, and S. G. Krasnich, in: *Research on Drying and Heat Processes* [in Russian], Nauka i Tekhnika, Minsk (1968).
5. A. S. Ginsburg and V. A. Rezhnikov, Drying of Food Products in a Fluidized Layer [in Russian], *Pishchevaya Promyshlennost'*, Moscow (1966).
6. I. F. Pikus, in: *Heat and Mass Transfer in Disperse Systems* [in Russian], Nauka i Tekhnika Minsk (1965).
7. V. A. Chlenov and N. V. Mikhailov, Drying of Free-Flowing Materials in a Vibrofluidized Layer [in Russian], Stroiizdat, Moscow (1967).
8. V. A. Sheiman, in: *Heat and Mass Transfer* [in Russian], Vol. 6, Part 1, Naukova Dumka, Kiev (1968).
9. A. V. Lykov, *The Theory of Drying* [in Russian], Gosenergoizdat, Moscow - Leningrad (1950).
10. A. V. Lykov, V. A. Sheiman, P. S. Kuts, and L. S. Slobodkin, *Inzh.-Fiz. Zh.*, 13, 5 (1967).
11. A. V. Lykov, V. A. Sheiman, and P. S. Kuts, in: *Heat and Mass Transfer* [in Russian], Vol. 6, Part 2, Naukova Dumka, Kiev (1968).